2002 Superbowl of High School Calculus

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SUPERBOWL OF HIGH SCHOOL CALCULUS

Remember:

1. Members of the same group may work together however they wish, but separate groups may not work together.
2. You may not use a calculator, computer, abacus, pile of smooth stones, or any other computing device of any type or level of technology.
3. Show all your work and be as neat as possible. Organization and presentation may serve as a tie-breaker to comparable solutions! Attach additional sheets as necessary.
4. Use a dark pencil or pen: you're faxing your answers, and I need to be able to read them!
5. Please simplify all answers as much as possible.
6. At 12 noon EST, it's all over but the crying (unless you have an authorized extension).
7. I'd like to include examples of great answers when I release the solutions.

   If you don't mind me using your work in the solution guide, check here: ☐. If you want me to include your school and team name if I use your answer, check here: ☐. If you don't want your answers used, check here: ☐.
8. The test administrator (teacher or staff member) should sign the bottom of this form when the test is over to indicate that the participants worked only in groups of 3, got no additional help, and did not use calculators, books or notes:

____

Test administrator's signature

All the Legal Stuff

Question 1: Alphabet soup

A continuous and differentiable polynomial function *f* is defined as follows:

$$f(x) = 2x^3 + ax^2 + bx + c\,.$$

a) Give the *x*-values representing locations where *f* may have relative extrema points.
b) Set up an equation whose solution is the *x*-value guaranteed by the Mean Value Theorem on the interval $[-1,1]$.
c) What conclusions, if any, can you draw about the concavity of *f* if you know that $a > 0$?

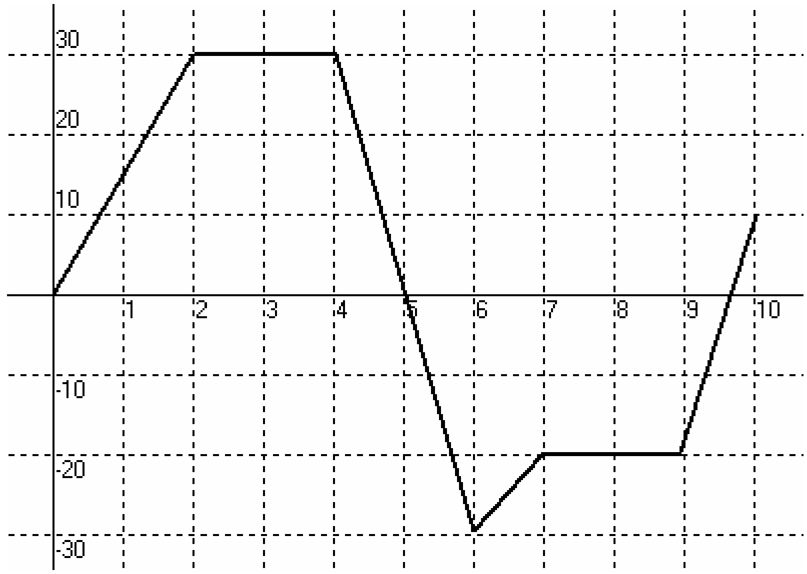
Question 2: Derivatives on parade

The functions *f*, *g*, *h*, *j*, and *k* are defined as follows:

$$f(x) = e^{2\ln x}$$
$$g(x) = \sin\left(\tan\left(2x\right)\right)$$
$$h(x) = x^{2/3}\left(x + \sqrt[4]{x}\right)$$
$$j(x) = \frac{e^x \cos x}{2 - x}$$
$$k(x) = 2x \cos x \sin x$$

Calculate $f'(2)$, $g'(\pi)$, $h'(1)$, $j'(0)$, and $k'(\pi)$, and rank them in order, from *least* to *greatest*. Make sure to show all of your work as you derive each.

Question 3: The bug problem



This graph depicts the *velocity* (measured in feet per minute) of a particularly ugly bug crawling back and forth along the x-axis for the first ten minutes of its determined, but hardly awe-inspiring, bug walk. Use this graph to answer the following questions, using correct units:

a) What was the bug's velocity and acceleration at $t = 1$ minute?

b) Assuming that the bug began its trek at the origin at exactly 10:13 AM, at what time(s) did the bug change direction? Your answers should be exact times, including seconds if appropriate.

c) Assuming, again, that the bug started at the origin, give the location of the bug at $t = 10$ minutes.

d) Give the average velocity and acceleration of the bug on the interval $0 \le t \le 10$.

e) At what time(s) is the bug the furthest from its starting point? Explain your answer.

Question 4: I trust you implicitly

The following questions refer to the equation $x^2 - 2xy + 3y^2 = 1$:

a) Sketch the graph of the equation

b) Calculate the general derivative, $\frac{dy}{dx}$, of the equation.

c) Write the equation of the tangent line to the graph at the point $\left(1, \frac{2}{3}\right)$.

d) Give the equation(s) of the graph's asymptote(s).

Question 5: Differential strokes for differential folks

I define the derivative, $h'(x)$, of some function $h(x)$ as follows: $h'(x) = \frac{x^2}{x^2+1}$.

a) Calculate $\int_0^{\pi/4} h'(x)\,dx$.

b) If $h(1) = -1$, find the equation representing $h(x)$.

c) Discuss the domain and range of $h(x)$.

Question 6: Piecewise of Cake

Here's a funky piecewise-defined function for you:

$$g(x) = \begin{cases} 2x^3 + x^2 - 3, & x < -1 \\ -\sqrt{x+k}, & x \geq -1 \end{cases}$$

Now, here are a few questions based on that ugly monstrosity:

a) Find the real value of k which makes $g(x)$ continuous.
b) Is $g(x)$ differentiable at $x = -1$? Justify your answer mathematically.
c) Consider the region R which is bounded by the lines $y = 0$, $x = -2$, $x = 2$, and the function you created in part (a) by finding the appropriate k value. What vertical line splits R into two equal regions?
d) If I define a new function $m(x) = \int_x^{-2} g(x)\, dx$, evaluate $m(2)$.

Question 7: Bored of Education

I have the unfortunate delight of hosting a calculus course during the first period of the school day. For many reasons (among them part-time jobs, a major term paper deadline, and an ultra-hip *Hanson* concert the night before) most of my students have a hard time staying awake in class. (I'm *sure* it has *nothing* to do with the way I teach ....) Out of curiosity, one day I pay close attention to the number of my 30 students who are actually paying attention during the 45-minute class period, and I compiled my results in the below chart:

Elapsed class time (in minutes)	5	10	15	20	25	30	35	40	45
Number of students awake	29	26	22	17	14	11	19	24	28

a) The number of students awake is a function of t, which we'll call it $A(t)$. Using the data in the table, approximate $A'(15)$ and explain the method you used. What does this value represent in the context of the problem?

b) Use your answer from part (a) to write the equation of the tangent line at $t = 15$.

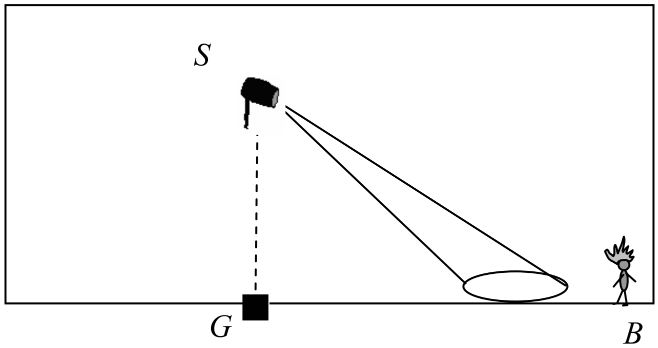
c) Use your answer from part (b) to estimate the number of students awake at time $t = 18$ minutes. Answer appropriately.

d) Calculate the slope of the line connecting (5,29) and (45,28) and explain what its value represents in the context of the problem.

e) Use the trapezoidal method (with trapezoids of width 5) to approximate the average number of students paying attention on the time interval [5,45].

f) **Bonus:** Create a continuous, differentiable function which models the number of students awake at any time t. The team with the best model receives an extra 5 points.

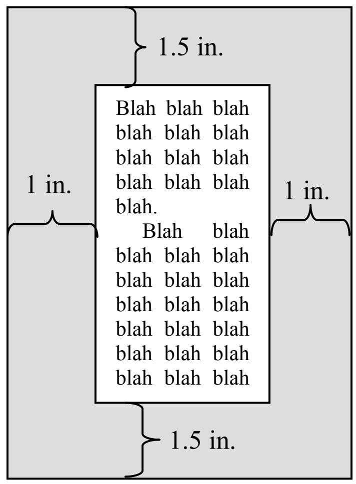
Question 8: Jailbreak!



Sideshow Bob is making a break for it and once again attempting to escape from the Springfield Penitentiary. It's all part of his diabolical plan to, once and for all, dispose of his arch-rival, Bart Simpson. The prison searchlight is located at point *S*, 500 feet directly across from the only entrance to the prison, the main gate at point *G*. The searchlight makes two complete revolutions per minute. How fast is the spotlight moving along the wall containing the gate (in feet per minute) when it hits Bob (at point *B*) if Bob is 1200 feet from the gate?

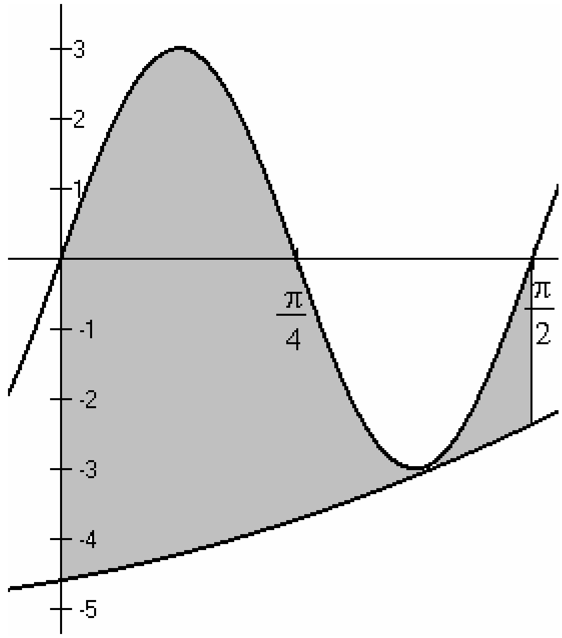
Question 9: A Harry dilemma

I am printing a book, which I hope will compete with the fabulously popular *Harry Potter* series. It's about a boy named Cedric that gets into all kinds of misadventures after he finds out he's very talented at ceramics. As he grows older, however, and passes through puberty, he becomes a very hairy potter indeed. Below is a typical page of text in the book.



I want the rectangular block containing text to measure 60 in^2, and I insist that the left and right margins are each 1 inch, whereas the top and bottom margins each measure 1.5 inches. What are the dimensions representing the minimum page size I can use to print my book while still conforming to my specifications?

Question 10: Things are a little shady



This shaded region above is defined on the interval $\left[0, \frac{\pi}{2}\right]$ and is bounded by the functions $f(x) = 3\sin 4x$ and $g(x) = \frac{2}{5}(x+1)^2 - 5$. Note that *f* and *g* do not intersect anywhere on that interval. Answer the following questions based on the diagram:

a) What is the area of the shaded region?

b) Set up (but do not integrate) an expression that represents the volume of a solid whose base is the shaded region and whose cross sections are semi-ellipses of height 5 perpendicular to the *x*-axis.

c) Set up (but do not integrate) an expression representing the volume of the solid which results when the shaded region is rotated about the line $y = -5$.

d) Explain how you could find the *x* value on the interval $\left[0, \frac{\pi}{2}\right]$ at which the distance between the two graphs is the largest.

Solutions and Grading Rubric for 2002 Superbowl of High School Calculus

The following pages represent the general guidelines for the grades assigned in the 2002 Superbowl of High School Calculus. Please note that the judge may exercise some judgment in determining if and how well each criterion is met. In general, the closer you were to the right idea, the more leniencies you were allowed.

1. **Alphabet Soup**

(a) Relative extrema points exist on *f(x)* wherever $f'(x)=0$:

$$f'(x)=6x^2+2ax+b=0$$

1 point: Derivative
1 point: Quadratic formula
1 point: Correct
1 point: Simplify

In order to solve this, you should use the quadratic formula:

$$x=\frac{-2a\pm\sqrt{4a^2-24b}}{12}$$
$$=\frac{-a\pm\sqrt{a^2-6b}}{6}$$

(b) In other words, where is the derivative equal to the average rate of change on [–1,1]?

$$f'(x)=\frac{f(1)-f(-1)}{1-(-1)}$$
$$6x^2+2ax+b=\frac{(2+a+b+c)-(-2+a-b+c)}{2}$$
$$6x^2+2ax+b=\frac{4+2b}{2}$$
$$6x^2+2ax+b=2+b$$
$$3x^2+ax=1$$

1 point: Equation
1 point: Right side
1 point: Correct
1 point: Simplify

Don't forget to simplify!

(c) Concavity questions must be directed to the second derivative:

$$f''(x)=12x+2a=6x+a=0$$

The curve will be concave down on the interval $\left(-\infty,-\frac{a}{6}\right)$ and concave up on $\left(-\frac{a}{6},\infty\right)$.
Clearly, the curve is concave up for all $x\geq 0$, since $12x+2a$ has to be positive if *a* is positive. Since *f* is a subic, it only has one point of inflection.

1 point: Answer
1 point: Explanation

2. **Derivatives on Parade**

(a)

$$f(x) = \left(e^{\ln x}\right)^2$$
$$f(x) = x^2$$
$$f'(2) = 2^2 = 4$$

(b)

$$g'(x) = 2\cos(\tan(2x)) \cdot \sec^2(2x)$$
$$g'(\pi) = 2\cos(\tan(2\pi)) \cdot \sec^2(2\pi)$$
$$g'(\pi) = 2(\cos 0) \cdot 1 = 2$$

(c)

$$h(x) = x^{5/3} + x^{11/12}$$
$$h'(x) = \frac{5}{3}x^{2/3} + \frac{11}{12x^{1/12}}$$
$$h'(1) = \frac{5}{3} + \frac{11}{12} = \frac{31}{12}$$

(d)

$$j'(x) = \frac{(2-x)\left(-e^x \sin x + e^x \cos x\right) - e^x \cos x(-1)}{(2-x)^2}$$
$$j'(0) = \frac{2 \cdot (0+1) + 1}{4} = \frac{3}{4}$$

(e)

$$k'(x) = 2x\cos x\,(\sin x)' + 2x(\cos x)'\sin x + (2x)'\cos x \sin x$$
$$k'(x) = 2x\cos^2 x - 2x\sin^2 x + 2\cos x \sin x$$
$$k'(\pi) = 2\pi - 0 + 0 = 2\pi$$

Correct order: $\frac{3}{4}$, 2, $\frac{31}{12}$, 4, 2π

2 points: Each derivative
–1 point: Incorrect order
–2 points: No order given

3. **The Bug Problem**

(a) Velocity comes right from the graph (since it's a velocity graph): 15 ft/min. Acceleration equals slope of segment connecting (0,0) and (2,30): 15 ft/min^2

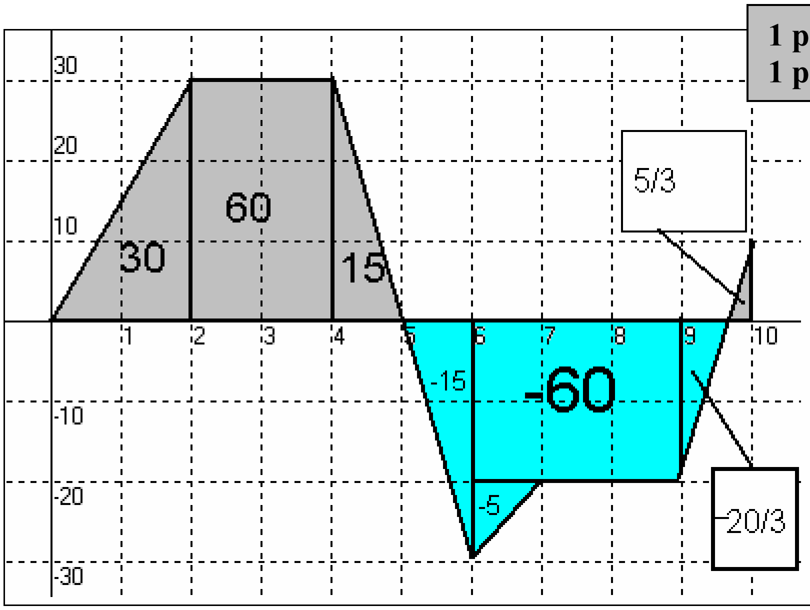
1 point: Velocity (½ units)
1 point: Acceleration (½ units)

(b) The bug changes direction whenever the graph passes through the x-axis, at 10:18 exactly and again at 10:22:40 (i.e. 10:22 and 40 seconds).

1 point: Time #1
1 point: Time #2 with seconds

(c) Find the area between the curve and the x-axis for that entire time: 20 feet to the right of the origin.

1 point: Correct area
1 point: Answer as position



(d)

$$\text{Average Velocity} = \frac{1}{10-0}\cdot\int_0^{10} v(t)\,dt$$
$$= \frac{1}{10}\cdot 20 = 2 \text{ ft/min}$$

1 point: Avg velocity (½ units)
1 point: Avg accel (½ units)

Average Acceleration = slope of segment connecting (0,0) and (10,10)

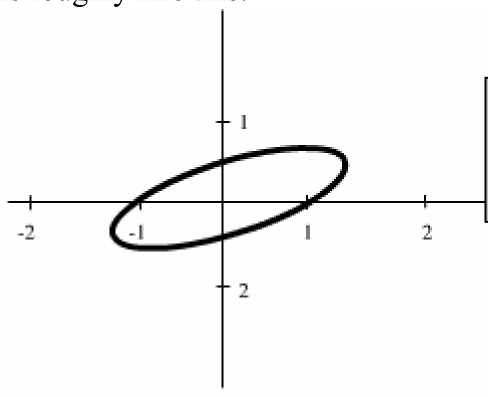
$$= \frac{10-0}{10-0} = 1 \text{ ft/min}^2$$

(e) At time $t = 5$ minutes, since $\int_0^5 v(t)\,dt$ is the largest integral of form $\int_0^c v(t)\,dt$, where $0 < c \le 10$.

1 point: Answer
1 point: Explanation

4. **I Trust You Implicitly**

(a) The graph looks roughly like this:



1 point: Ellipse graph
1 point: Slanty graph
1 point: $-2 < x < 2$

The best way to draw it is to plug values of x and find the corresponding y values. If you draw a slope field, that can also help you see that it's a slanted ellipse, but slope fields are not yet part of the AB curriculum, so they're not repuired.

(b)

$$2x - \left(2x\frac{dy}{dx} + 2y\right) + 6y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(-2x + 6y) = -2x + 2y$$

$$\frac{dy}{dx} = \frac{y - x}{3y - x} = \frac{x - y}{x - 3y}$$

2 points: Implicit differentiation
1 point: Correct

(c) Slope at that point is $\frac{dy}{dx} = \frac{1 - \frac{2}{3}}{1 - 2} = \frac{\frac{1}{3}}{-1} = -\frac{1}{3}$. Therefore, the line has the following equations (any is correct):

$$y - \frac{2}{3} = -\frac{1}{3}(x - 1)$$

$$3y - 2 = -x + 1$$

$$x + 3y = 3$$

$$y = -\frac{1}{3}x + 1$$

2 points: Linear equation

(d) There are no asymptotes on an ellipse.

1 point: None
1 point: Because it's an ellipse

5. **Differential Strokes for Differential Folks**

(a)

$$\int_0^{\pi/4} \frac{x^2}{x^2+1}\,dx$$
$$= \int_0^{\pi/4} \frac{x^2+1-1}{x^2+1}\,dx$$
$$= \int_0^{\pi/4} \frac{x^2+1}{x^2+1}\,dx - \int_0^{\pi/4} \frac{1}{x^2+1}\,dx$$
$$= \int_0^{\pi/4} dx - \int_0^{\pi/4} \frac{1}{x^2+1}\,dx$$
$$= x \,\Big|_0^{\pi/4} - \arctan x \Big|_0^{\pi/4}$$
$$= \frac{\pi}{4} - \arctan\frac{\pi}{4}$$

3 points: Integral set-up & work
1 point: Presence of arctangent
1 point: Correct answer

Note that $\arctan\frac{\pi}{4} \neq 1$! Instead, $\arctan 1 = \frac{\pi}{4}$.

(b) To find $h(x)$, integrate $h'(x)$ as you did in part (a) and plug in the known values:

$$\int \frac{x^2}{x^2+1}\,dx = x - \arctan x + C$$

$$1 - \arctan 1 + C = -1$$
$$C = -2 + \frac{\pi}{4}$$

1: Correct answer
1: Plugged correctly into solution to part (a)

Therefore, $h(x) = x - \arctan x - 2 + \frac{\pi}{4}$.

(c) The domain is clearly $(-\infty, \infty)$, as all terms in h share this domain. (We can be sure that this is arctangent's domain since it is tangent's range). As far as range is concerned, the presence of the x term ensures that the range will also be $(-\infty, \infty)$, as arctangent's value will fall in the interval $-\frac{\pi}{2} \leq \theta < \frac{\pi}{2}$.

1: Correct domain
1: Correct range
1: Good justification for both

Note: Although I used the notation "arctan x," the notation "$\tan^{-1} x$" is also acceptable.

6. **Piecewise of cake**

2: Correct set-up & work
1: Correct answer

(a) For $g(x)$ to be continuous:

$$2(-1)^3+(-1)^2-3=-\sqrt{-1+k}$$
$$-4=-\sqrt{-1+k}$$
$$k-1=16$$
$$k=17$$

(b) Since $\frac{d}{dx}\left(2x^3+x^2-3\right)\neq\frac{d}{dx}\left(-\sqrt{x+17}\right)$ when evaluated at $x=-1$, g is not differentiable there:

$$6(-1)^2+2(-1)\neq-\frac{1}{2\sqrt{-1+17}}$$
$$4\neq-\frac{1}{8}$$

1: Correct answer
1: Correct justification
1: Derivatives worked out

(c) First, find the complete area:

$$\text{Area}=\int_{-2}^{-1}\left(2x^3+x^2-3\right)dx+\int_{-1}^{2}\left(-\sqrt{x+17}\right)dx$$
$$=\left(\frac{x^4}{2}+\frac{x^3}{3}-3x\right)\Bigg|_{-2}^{-1}+\left(-\frac{2}{3}(x+17)^{3/2}\right)\Bigg|_{-1}^{2}$$
$$=\left(\frac{1}{2}-\frac{1}{3}+3-8+\frac{8}{3}-6\right)-\frac{2}{3}\left(19\sqrt{19}-64\right)$$
$$=\left(-\frac{49}{6}\right)+\left(\frac{128}{3}-\frac{38}{3}\sqrt{19}\right)$$
$$=\frac{69}{2}-\frac{38}{3}\sqrt{19}$$

1: Correct set-up of whole area
1: Each separate integral is worth a half point
1: Correct form for final answer (i.e. half of whole area)

Because of the complexity of the calculations, I will accept an equation whose solution is the value in question for full credit:

$$\int_{-2}^{-1}\left(2x^3+x^2-3\right)dx+\int_{-1}^{c}\left(-\sqrt{x+17}\right)dx=\frac{1}{2}\left(\frac{69}{2}-\frac{38}{3}\sqrt{19}\right)$$

(d) This is much easier than it looks—all the variables are x! Since $g(2)=-\sqrt{2+17}=-\sqrt{19}$:

$$m(2)=-\int_{-2}^{2}-\sqrt{19}\,dx=\sqrt{19}\cdot\int_{-2}^{2}dx=4\sqrt{19}$$

1: Correct answer

7. **Bored of Education**

(a) Approximate it by calculating the slope connecting (10,26) and (15,22) or (15,22) and (20,17). Alternately, you can use the slope between (10,26) and (20,17):

1: One of these slopes
1: Correct interpretation

$$\frac{22-26}{15-10}=-\frac{4}{5} \text{ or } \frac{17-22}{20-15}=-1 \text{ or } \frac{17-26}{20-10}=-\frac{9}{10}$$

This represents the rate at which students are waking up. Since it's negative, it shows that students are falling asleep at the rate of about 1 student per minute.

(b)

$$y-22=-\frac{4}{5}(x-15) \qquad y-22=-1(x-15) \qquad y-22=-\frac{9}{10}(x-15)$$

$$y=-\frac{4}{5}x+34 \qquad y=-x+37 \qquad y=-\frac{9}{10}x+\frac{71}{2}$$

$$4x+5y=170 \qquad x+y=37 \qquad 9x+10y=355$$

1: Uses slope from (a)
1: Correct equation

(c)

$$y=-\frac{4}{5}(18)+34\approx 19 \text{ or } 20 \qquad y=-18+37=19 \qquad y=-\frac{9}{10}(18)+\frac{71}{2}\approx 19$$

An appropriate answer is an integer, since there is no such thing as 19.6 students.

1: Correct answer
1: Integer

(d) The slope is $\frac{28-29}{45-5}=-\frac{1}{40}$ students/minute. This is the average rate at which students were waking up on the time interval [5.45]. Since it's negative, it means that, on average students were falling asleep.

1: Correct slope
1: Correct interpretation

(e) First use the Trapezoidal Rule to approximate $\int_5^{45} A(t)\,dt$:

$$\frac{45-5}{2(8)}(29+2\cdot 26+2\cdot 22+2\cdot 17+2\cdot 14+2\cdot 11+2\cdot 19+2\cdot 24+28)$$

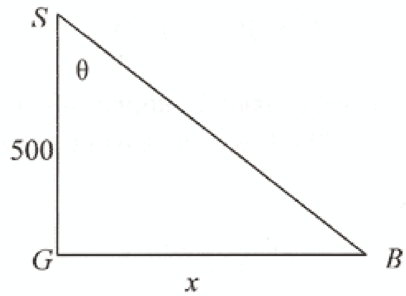
$$\frac{5}{2}(323)=\frac{1615}{2}$$

1: Correct area
1: Average value given

The average value of $A(t)$ is $\frac{1}{40}\cdot\frac{1615}{2}=\frac{1615}{80}\approx 20$ students.

8. **Jailbreak!**

Note: This problem, as stated in the original version of the contest, did not specify that you were finding the speed of the spotlight as it moved along the wall, as the problem now states. Therefore, I gave credit to solutions finding other speeds as long as they were explained and had mathematical validity.



Since we know the speed of the rotating light, $\frac{d\theta}{dt} = 2 \cdot 2\pi = 4\pi$ radians/minute. We are looking for $\frac{dx}{dt}$, and we note from the diagram that:

2: *dθ/dt* found
2: Tangent statement
2: Tangent Derived correctly

$$\tan\theta = \frac{x}{500}$$
$$x = 500\tan\theta$$
$$\frac{dx}{dt} = 500\sec^2\theta\,\frac{d\theta}{dt}$$

When Bob is 1200 feet from the gate, we have a 5-12-13 right triangle, so $\overline{SB}$ has length 1300, and $\sec\theta = \frac{1300}{500} = \frac{13}{5}$:

2: Length 1300 found
1: Values substituted correctly
1: Correct answer with units

$$\frac{dx}{dt} = 500\left(\frac{13}{5}\right)^2 \cdot 4\pi$$
$$= 2000\pi \cdot \frac{169}{25}$$
$$= 13520\pi \text{ ft/min} \approx 42474.3 \text{ ft/min}$$

9. **A Harry Dilemma**

If I set w = text width, and l = text length, then I know $wl = 60$ and $l = \frac{60}{w}$. Since I am minimizing the dimensions of the *page*, not the text, I want to maximize the equation:

$$D = (w+2)(l+3)$$

This is because I need to figure in the margins, 2 total inches for the width and 3 total inches for the length $(2 \cdot 1.5 \text{ inches})$. Now, plug in out new l and differentiate that equation:

$$D = (w+2)\left(\frac{60}{w}+3\right)$$
$$D = 60 + 3w + 120w^{-1} + 6$$
$$D' = 3 - \frac{120}{w^2}$$

2: Eliminate a variable
2: Correct optimization equation
3: Equation derived correctly

Set the derivative equal to 0 to find critical points:

$$\frac{120}{w^2} = 3$$
$$3w^2 = 120$$
$$w^2 = 40$$
$$w = 2\sqrt{10}$$

2: 1 each for correct dimensions
1: Justify that it's a minimum

Therefore, the page width should be $2\sqrt{10}+2$ inches. Ensure it's a minimum with a wiggle/sign graph or the Second Derivative Test:

$$D'' = \frac{240}{w^3}$$

Clearly, D'' will be positive since $2\sqrt{10}+2$ is positive, meaning that this represents a relative minimum. To finish, find the length that corresponds to $2\sqrt{10}+2$:

$$\text{page length } = \frac{60}{w} + 3$$
$$= \frac{60}{2\sqrt{10}} + 3$$
$$= 3\sqrt{10} + 3 \text{ or } \frac{30}{\sqrt{10}} + 3 \text{ inches}$$

10. **Things are a Little Shady**

(a)

$$\int_0^{\pi/2}\left(3\sin 4x-\left(\frac{2}{5}(x+1)^2-5\right)\right)dx$$

$$=3\int_0^{\pi/2}\sin 4x\,dx-\frac{2}{5}\int_0^{\pi/2}(x+1)^2\,dx+5\int_0^{\pi/2}dx$$

$$=-\frac{3}{4}(\cos 4x)\Big|_0^{\pi/2}-\frac{2}{5}\left(\frac{x^3}{3}+x^2+x\right)\Bigg|_0^{\pi/2}+5(x)\Big|_0^{\pi/2}$$

$$=-\frac{3}{4}(1-1)-\frac{2}{5}\left(\frac{\pi^3}{24}+\frac{\pi^2}{4}+\frac{\pi}{2}\right)+5\left(\frac{\pi}{2}\right)$$

$$=-\frac{\pi^3}{60}-\frac{\pi^2}{10}-\frac{\pi}{5}+\frac{5\pi}{2}=\frac{-\pi^3-6\pi^2+138\pi}{60}$$

1: Integral and boundaries correct
2: Correct antiderivatives
1: Correct answer
1: Simplified (one fraction!)

(b) Ellipses have area πab, so a semi-ellipse has area $\frac{\pi ab}{2}$:

$$\text{Volume}=\frac{5\pi}{2}\int_0^{\pi/2}\left(3\sin 4x-\frac{2}{5}(x+1)^2+5\right)dx$$

1: Cross section formula correctly used
1: Area of semi-ellipse formula used correctly

(c) Use the Washer Method:

$$\pi\int_0^{\pi/2}\left[(3\sin 4x+5)^2-\left(\frac{2}{5}(x+1)^2\right)^2\right]dx$$

2: Washer Method used correctly

(d) To find the spot where the distance between them is the largest, find the absolute maximum of the equation $d(x)=3\sin 4x-\left(\frac{2}{5}(x+1)^2-5\right)$ using the Extreme Value Theorem on the interval $\left[0,\frac{\pi}{2}\right]$. *Note: On the original version of the contest, this part asked for an integral expression in the answer, which is not necessary, so it was not included in the grading.*

1: Correct answer & explanation

Statistical Analysis for 2002 Superbowl Questions

In the chart that follows are the means and medians for each individual question. In addition, I have denoted how many teams (of the 40 that participated) scores on the ranges of 7-10+, 4-6, 1-3 and 0. (Each problem was worth a maximum of 10 points.) A team may earn more than 10 points for a question by earning *Best Plays*, which are 2-point bonuses given to one team per question, representing the best answer given by all competing teams for that question.

Question	Mean	Median	Score of 7-10+	Score of 4-6	Score of 1-3	Score of 0
1. Alphabet Soup	5.42	5.88	16	10	7	7
2. Derivatives on Parade	7.56	8.50	25	11	2	2
3. The Bug Problem	5.77	6.00	17	14	8	1
4. I Trust You Implicitly	5.10	5.00	12	17	8	3
5. Differential Strokes	2.11	0.00	4	5	10	21
6. Piecewise of Cake	3.19	2.25	5	12	8	15
7. Bored of Education	5.48	6.25	19	8	5	8
8. Jailbreak!	4.29	4.00	12	10	4	14
9. A Harry Dilemma	4.65	5.00	15	9	3	13
10. Things are Shady	3.64	3.50	7	12	13	8